

# Mental Calculation Progression



## Summer 2022

**The National Curriculum for mathematics aims to ensure that all pupils:**

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

This document was written in response to the heightened demands of the National Curriculum (2014). It aims to support teachers with a map of progression in mental fluency. The programme of study includes references to mental calculation but lacks the detail needed to provide a coherent pathway. It is up to schools to decide upon what this should look like. This guidance document provides the necessary detail.

## Teaching children to calculate mentally

The ability to calculate in your head is an important part of mathematics. It is also an essential part of coping with society's demands and managing everyday events.

**This progression aims to:**

- list the number facts that children are expected to recall rapidly
- identify the mental methods that might be taught to children to help them to calculate accurately and efficiently
- set out expectations for the types of calculations that children should be able to do mentally

Practice is a key approach to developing the automaticity needed to reduce cognitive load. Pupils who have facts and skills at their fingertips are more likely to attend to the particulars of new learning than those that do not. These pupils have to work harder and are over-burdened. Practice not as meaningless repetition of facts in which pupils chant without thought or as a series of isolated facts learnt at home then tested in school, but as a chance to rehearse them within exercises that develop better thinking. Practice is an opportunity to keep facts and skills 'simmering' and a further chance to vary the ways that they are presented. Schools should be mindful of the *quality* of practice rather than the *quantity*. Similarly, they are advised to focus upon the facts and skills that will make the greatest difference to mental fluency at each phase.

## Factual Fluency Guidance

*“Pupils who are not able to quickly and easily recall maths facts struggle with calculations due to their working memory being overloaded... Many young pupils need and benefit from systematic provision of sequenced core content that becomes the building blocks of later success.”*

OFSTED Mathematics Review, May 2021

## Factual Fluency – Additive Facts

The full set of additional calculations that pupils need to be able to solve with automaticity are shown in the table below. Pupils must also be able to solve the corresponding subtraction calculations with automaticity.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

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Pupils must be fluent in these facts by the end of Year 2 and should continue with regular practice through Year 3 to secure and maintain fluency.

The progression table below summarises the order in which pupil should learn these **additive** number facts.

	Year 1	Year 2	Year 3	Year 4	Year 5
Additive factual fluency	Addition and subtraction within 10.	Addition and subtraction across 10.	Secure and maintain fluency in addition and subtraction within and across 10, through continued practice.		

## The importance of focusing on fluency in addition and subtraction facts

This defined set of addition and subtraction facts builds the basis of all additive calculation, just as times tables are the building blocks for all multiplicative calculation.

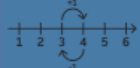



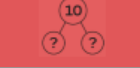







If children are not fluent in these facts, then when they are solving more complex problems, the working memory is taken up by calculating basic facts, and children have less working memory to

focus on solving the actual problem. So fluency in basic facts allows children to tackle more complex maths effectively.

<https://numbersensematics.com/media/2178/article-ncetm-magazine.pdf>

## The Number Sense Approach

The Number Facts Fluency Programme teaches a core set of number facts. Modelled on the phonics programmes used in early reading, the programme groups the grid facts and teaches them systematically alongside the calculation strategies that can be used to solve them. Below is an explanation of the strategies taught:

<b>One More, One Less</b> 	When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. $5 - 1 = 4$ ).	<b>Number Neighbours: Spot the Difference</b> 	Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2.  Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. $5 - 4 = 1$ ), of adjacent odds (e.g. $9 - 7 = 2$ ) or adjacent evens (e.g. $6 - 4 = 2$ ).
<b>Two More, Two Less: Think Odds and Evens</b> 	If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.	<b>7 Tree and 9 Square</b> 	Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that $7 - 3 = 4$ . Visualising the 9 square helps remember that $3 + 6 = 9$ .
<b>Number 10 Fact Families</b> 	Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.	<b>Ten and A Bit</b> 	The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. $3 + 10 = 13$ , $17 - 7 = 10$ , $12 - 10 = 2$ ).
<b>Five and A Bit</b> 	The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. $5 + 3 = 8$ , $9 - 5 = 4$ ).	<b>Make Ten and Then</b> 	Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. $8 + 6$ can be calculated by thinking ' $8 + 2 = 10$ and 4 more makes 14'). The same strategy can be applied to subtractions through 10.
<b>Know about 0</b> 	When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.	<b>Adjust It</b> 	Any addition and subtraction can be calculated by adjusting from a fact you know already. (e.g. $6 + 9$ is one less than $6 + 10$ ).
<b>Doubles and Near Doubles</b> 	Memorise doubles of numbers to 10, using a visual approach. Then use these known double facts to calculate near doubles and hidden doubles. Once we know $6 + 6 = 12$ then $6 + 7$ and $5 + 7$ is easy.	<b>Swap It</b> 	When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. $1 + 8 = 8 + 1$ . Sometimes reversing the order of the two addends makes addition easier to think about conceptually.



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## Mastering Number Approach

This programme focuses on the key knowledge and understanding needed in Reception classes, and progression through KS1. There is an expectation that schools will provide a daily teaching session for all children of 10 to 15 minutes, in addition to their normal maths lesson. The aim over time is that children will leave KS1 with fluency in calculation and a confidence and flexibility with number. Attention is given to key knowledge and understanding needed in Reception classes, and progression through KS1 to support success in the future.

<https://www.ncetm.org.uk/maths-hubs-projects/mastering-number/>

### **The nature and aims of the mastering number**

- Develop automaticity in number facts
- Develop deep conceptual understanding of number relationships (number sense)

Strategies to achieve the above:

- ✓ Subitising (seeing numbers **without counting** and applying spatial reasoning)
- ✓ Composition of numbers (the numbers inside!)
- ✓ Seeing number relations through mathematical structure

- ✓ Embedding visual images which expose number relations
- ✓ Variation (particularly the 'not' property)
- ✓ Small-step learning trajectory
- ✓ Becoming mathematically observant – looking for number relationships, reasoning about them and generalising.

## **Factual Fluency – Multiplicative Facts**

*If multiplication facts are learnt and stored, rather than being calculated or by skip counting repeatedly, then they will require less activity from the brain, reducing the 'cognitive load' and essentially 'freeing up' space to focus brain activity on the application of the facts NOT the facts themselves.*

**Dehaene, S.** [http://win.pisavisionlab.org/teaching/burr/piazzadehaene\\_chapgazzaniga.pdf](http://win.pisavisionlab.org/teaching/burr/piazzadehaene_chapgazzaniga.pdf)

The full set of multiplication calculations that pupils need to be able to solve by automatic recall are shown in the table below. Pupils must also have automatic recall of the corresponding division facts.

1 × 1	1 × 2	1 × 3	1 × 4	1 × 5	1 × 6	1 × 7	1 × 8	1 × 9	1 × 10	1 × 11	1 × 12
2 × 1	2 × 2	2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 8	2 × 9	2 × 10	2 × 11	2 × 12
3 × 1	3 × 2	3 × 3	3 × 4	3 × 5	3 × 6	3 × 7	3 × 8	3 × 9	3 × 10	3 × 11	3 × 12
4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 1	5 × 2	5 × 3	5 × 4	5 × 5	5 × 6	5 × 7	5 × 8	5 × 9	5 × 10	5 × 11	5 × 12
6 × 1	6 × 2	6 × 3	6 × 4	6 × 5	6 × 6	6 × 7	6 × 8	6 × 9	6 × 10	6 × 11	6 × 12
7 × 1	7 × 2	7 × 3	7 × 4	7 × 5	7 × 6	7 × 7	7 × 8	7 × 9	7 × 10	7 × 11	7 × 12
8 × 1	8 × 2	8 × 3	8 × 4	8 × 5	8 × 6	8 × 7	8 × 8	8 × 9	8 × 10	8 × 11	8 × 12
9 × 1	9 × 2	9 × 3	9 × 4	9 × 5	9 × 6	9 × 7	9 × 8	9 × 9	9 × 10	9 × 11	9 × 12
10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12

Pupils must be fluent in these facts by the end of year 4, and this is assessed in the multiplication tables check. Pupils should continue with regular practice through year 5 to secure and maintain fluency.

The 36 most important facts are highlighted in the table. Fluency in these facts should be prioritised because, when coupled with an understanding of commutativity and fluency in the formal written method for multiplication, they enable pupils to multiply any pair of numbers.

The **progression table below** summarises the order in which pupil should learn these multiplicative number facts. Pupils should learn the multiplication tables in the 'families' described in the progression table – making connections between the multiplication tables in each family will enable pupils to develop automatic recall more easily, and provide a deeper understanding of multiplication and division.

	Year 1	Year 2	Year 3	Year 4	Year 5
<b>Multiplicative factual fluency</b>			Recall the 10 and 5 multiplication tables, and corresponding division facts.	Recall the 3, 6 and 9 multiplication tables, and corresponding division facts.	Secure and maintain fluency in all multiplication tables, and corresponding division facts, through continued practice.
			Recall the 2, 4 and 8 multiplication tables, and corresponding division facts.	Recall the 7 multiplication table, and corresponding division facts.	
				Recall the 11 and 12 multiplication tables, and corresponding division facts.	



# Times Tables Challenge Approach

## Maths Challenge – What is it?

- Systematic, whole class approach to learning the times tables.
- Aims to break down the learning of the times tables into manageable chunks learning a times table at a time.
- Importance of the commutative law and the relationship with division facts.
- Rote learning in which children learn the number facts AND a learned sound pattern (this is important).

## Which year groups are involved?

- **Year 2:** Children should have an understanding of multiplying as repeated addition. e.g. 7 x 5 drawing images 7 groups of 5 drawn. Children should have a knowledge of the x2 – x5 – x10 times table.
- **Year 3:** Introduce Times Table Challenge to enable children to become fluent in (2x, 4x, 5x, 8x and 10x)
- **Year 4:** Revising Year 3 content and then children learn (3x, 6x, 7x, 9x, 11x and 12x)

## What are the Times Table Challenge booklets?

The booklets should be worked through in the following order, to match the order suggested in the National Curriculum Guidance (July 2020)

Booklet A: 10 times table	Booklet F: 3 times table
Booklet B: 5 times table	Booklet G: 6 times table
Booklet C: 2 times table	Booklet H: 9 times table
Booklet D: 4 times table	Booklet I: 7 times table
Booklet E: 8 times table	Booklet J: 11 times table
	Booklet K: 12 times table

Within each booklet there are 22 tests, ordered as follows:

- Tests 1 – 4: First half of the new times table
- Tests 5 – 8: Second half of the new times table
- Tests 9 – 12: All the new times table
- Tests 13 – 22: The new times table combined with previously learnt times tables.

There are two exceptions to this, the 10 times and 11 times table booklets. As these are quicker for children to learn, all the facts are introduced at once rather than split into 'first half' and 'second half' of the times table.

It is important that you work through the booklets in the order provided in the table above, otherwise the children will meet facts in tests 13 – 22 that they have not yet learnt.

The National Curriculum Guidance explains that the facts it is essential to master in Year 4 to be ready to progress to Year 5 are the facts up to  $9 \times 9$ , as these facts are the ones that occur as within column calculations in formal written methods. Therefore, Booklets B – I include facts with multipliers of 2 – 9 only.

Times tables facts with a factor of 11 and 12 are only introduced in the final 2 booklets, so that most of the time can be spent learning the most essential facts. However, you should aim to complete all the booklets so that secure in all times tables facts prior to the Year 4 check. Facts with a multiplier of 0 and 1 are not included, as these do not need to be learnt in the same way as other facts.

The 10 times table is of course also essential for progression, and this is learnt in booklet A, and then included in tests 13 – 22 in each of the subsequent booklets.

About 20% of the facts are expressed as division facts, to give children practice deriving division facts from learnt multiplication facts.

### How much time is involved?

- Little and often focus on learning times tables is best. The aim of the Times Table Challenge is that it fits into a 5-7minute slot (so twice a day is only 10-14 minutes of time).
- A two-minute times table test, twice a day. Each test comprises of 40 questions in each test (on average of 3 seconds per question).
- Each test will begin with a counting stick activity to support the children with learning the facts. It's critical that children are actively learning the facts and not just being tested on them

### How do we introduce a new times table?

- It is important to highlight what the children already know as known facts. Through the knowledge of commutative law, they can really see even at this stage how much they already know.

**My Times Table  
Practice Booklet  
6 Times Tables**

Name: \_\_\_\_\_  
 Class: \_\_\_\_\_

New facts in this booklet:  
 6 x 6 = 36  
 7 x 6 = 42  
 8 x 6 = 48  
 9 x 6 = 54

1		2	
6 x 5 =	6 x 6 =	6 x 3 =	3 x 6 =
36 ÷ 6 =	24 ÷ 6 =	2 x 6 =	30 ÷ 6 =
4 x 6 =	6 x 2 =	30 ÷ 5 =	4 x 6 =
12 ÷ 6 =	6 x 4 =	6 x 2 =	36 ÷ 6 =
6 x 2 =	4 x 6 =	6 x 2 =	5 x 6 =
6 x 3 =	30 ÷ 5 =	5 x 6 =	2 x 6 =
5 x 6 =	2 x 6 =	6 x 5 =	6 x 5 =
4 x 6 =	6 x 5 =	12 ÷ 2 =	6 x 6 =
3 x 6 =	6 x 6 =	4 x 6 =	18 ÷ 3 =
18 ÷ 3 =	2 x 6 =	4 x 6 =	4 x 6 =
30 ÷ 6 =	6 x 5 =	3 x 6 =	6 x 2 =
6 x 4 =	6 x 5 =	24 ÷ 6 =	6 x 2 =
6 x 4 =	12 ÷ 2 =	3 x 6 =	6 x 6 =
6 x 3 =	5 x 6 =	2 x 6 =	6 x 4 =
6 x 3 =	5 x 6 =	6 x 3 =	12 ÷ 6 =
3 x 6 =	18 ÷ 6 =	6 x 5 =	6 x 6 =
2 x 6 =	6 x 5 =	5 x 6 =	6 x 4 =
24 ÷ 4 =	6 x 6 =	18 ÷ 6 =	24 ÷ 4 =
4 x 6 =	6 x 2 =	6 x 4 =	6 x 3 =
6 x 6 =	3 x 6 =	5 x 6 =	2 x 6 =

First part of 6 times table

- Write up the associated division facts alongside the times table facts so that the children can see the clear relationship between multiplication and division.
- Learn a fact at a time e.g. one a day.
- Introduce times tables alongside another activity e.g. counting stick, rolling numbers...

## Principles for Learning Multiplication Facts

1. Learn each number sentence as a memorised phrase by repeating the sound pattern out loud.
2. Learn each fact one way round only, then get confident at switching factors. Largest factor first!  $4 \times 6 =$  becomes six fours are twenty-four.

All of these facts are chanted  
four threes are twelve

$$\begin{array}{l} 4 \times 3 = 12 \\ 3 \times 4 = 12 \\ 12 \div 4 = 3 \\ 12 \div 3 = 4 \end{array}$$

If I know  $4 \times 3$  what other facts  
can I work out?

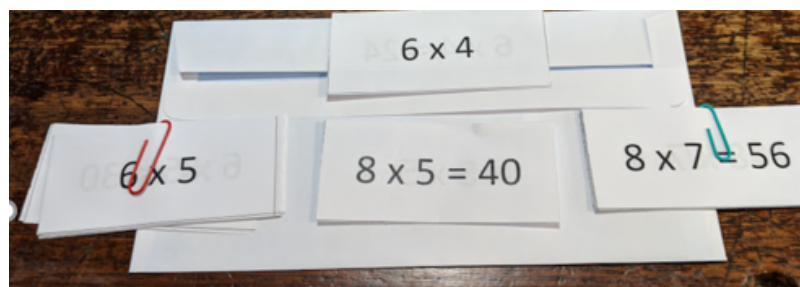
3. Don't think! (about the only time in maths when thinking is unhelpful!) When trying to recall a fact, say the WHOLE number sentence out loud and see if the answer trips off your tongue. If not, try the commutative and see if it comes then.
4. Leave the answers on the board.

## Reading out the answers

- The children mark their own booklets so that they can fill in any gaps if necessary.
- The full times table fact is read out. We always say the larger number first so that they are only learning one sound pattern for each fact. For example, if the number fact is  $6 \times 7 = 42$ , we say seven sixes are forty two.
- The children then repeat that fact back to you. It's important that every child does this.
- For division facts say the following. For 18 divided 3 say MMM threes are eighteen. The children then say the learnt times table fact. Six threes are eighteen.
- Once marked the children then share their results with the class and identify a number fact they need to learn.

## Additional support: The envelope system

- Individual 1-1 intervention for those children who are struggling to remember number facts.
- Guidance provided to parents as to how they can support the individual's learning.
- Start by conferencing the child to identify the number facts they can recall/known facts (green) and unknown facts (red). They then pick two **different** unknown facts and use them as a bookmark to self-test before reading.



Suggested Coverage Overview in Year 3 and 4

Maths Times Table Challenge Year 3 Annual Overview

	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S											
Sept Term 1	--- Summer Holiday ---					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30				
	Consolidate additive facts																																						
Oct	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	--- Half Term Holiday ---							
Nov Term 2				1	2	3	4	5	6	7	8	9	Introduce 10x														26	27	28	29	30								
																												Introduce 5x											
Dec						1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30				
																												--- Christmas Holiday ---											
Jan Term 3	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31							
	--- Christmas Holiday ---							Introduce 2x																															
Feb Term 4					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28							
								Introduce 4x															--- Half Term Holiday ---																
Mar					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Introduce 8x													30	31			
April Term 5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30									
								--- Easter Holiday ---							--- Easter Holiday ---																								
May			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2				
								Keep going with x IF NEEDED or move on to all																											--- Half Term Holiday ---				
June Term 6	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30											
	Testing of all 10x, 2x, 5x, 4x and 8x																																						
July	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	--- Summer Holiday ---																	

Maths Times Table Challenge Year 4 Annual Overview

	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S			
Sept Term 1	--- Summer Holiday ---				1	2	3	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30			
	Review Year 3 learning (10x, 5x, 2x, 4x and 8x)												Introduce 3x																									
Oct	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	--- Half Term Holiday ---						
Nov Term 2				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	Introduce 9x				
Dec						1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30			
Jan Term 3	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31						
	--- Christmas Holiday ---				Introduce 7x																																	
Feb Term 4					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28						
													Introduce 11x					--- Half Term Holiday ---																				
Mar					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31			
													Introduce 12x																									
April Term 5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30								
													--- Easter Holiday ---					--- Easter Holiday ---					Mixed x tables															
May			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2			
																												--- Half Term Holiday ---										
June Term 6	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30										
													Multiplication Tables Check Testing Period																									
July	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31							
																							--- Summer Holiday ---					--- Summer Holiday ---										

## **Principles of teaching mental calculation**

A feature of mental calculation is that a type of calculation can often be worked out in several different ways. Which method is the best will depend on the numbers involved, the age of the children and the range of methods that they are confident with.

Therefore, it is important to:

- teach a mental strategy explicitly but in addition invite children to suggest an approach and to explain their methods of solution to the rest of the class.

This has the advantages that:

- children get used to looking out for an approach they can call their own
- children doing the explaining, clarify their own thinking
- children who are listening, develop their awareness of the range of possible methods
- the activity can lead to a discussion of which methods are the most efficient.

Revisiting mental work daily and even devoting a whole lesson to it from time to time, helps children to generate confidence in themselves and a feeling that they control calculations rather than calculations controlling them. Regular short practice keeps the mind fresh. Mental calculation is one of those aspects of learning where – if you don't use it you will end up losing it!

- Commit regular time to teaching mental calculation strategies.
- Provide practice time with frequent opportunities for children to use one or more facts that they already know to work out more facts.
- Introduce practical approaches and jottings with models and images children can use to carry out calculations as they secure mental strategies.
- Engage children in discussion when they explain their methods and strategies to you and their peers.

## **Progression in mental calculation strategies**

### **Key addition and subtraction strategies**

- **Counting forwards and backwards** – in a variety of interval steps

$14 + 3$                       count on in ones from 14

$27 - 4$                       count on or back in ones from any two-digit number

$18 - 4$                       count back in twos from 18

$30 + 3$                       count on in ones from 30

- **Reordering** – know when and how to reorder to make calculations easier

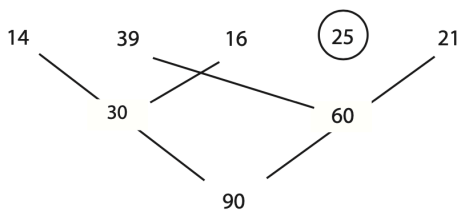
$$2 + 7 = 7 + 2$$

$$5 + 13 = 13 + 5$$

- **Finding complements** – identifying pairs or trios of 1, 10, 20 and 100...(like reordering)

$$14 + 39 + 16 + 25 + 21$$

it is sensible to pair numbers:

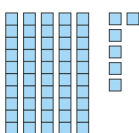
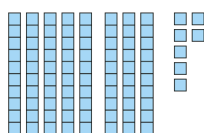


$$90 + 25 = 115$$

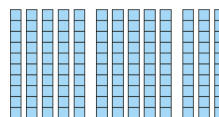
- **Partitioning (regrouping)** – the ability to break numbers up and recombine them flexibly. It is important that children are aware that numbers can be partitioned – both along the place value boundaries (canonically) and in other ways (non-canonically).

$$\begin{array}{r} 87 \\ / \quad \backslash \\ 80 \quad 7 \end{array} + \begin{array}{r} 56 \\ / \quad \backslash \\ 50 \quad 6 \end{array}$$

$$\begin{array}{r} 87 \\ / \quad \backslash \\ 80 \quad 7 \end{array} + \begin{array}{r} 56 \\ / \quad \backslash \\ 50 \quad 6 \end{array} = 130 + 13$$



then

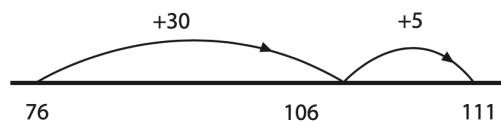


130

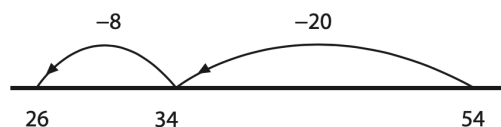


13

$$76 + 35:$$

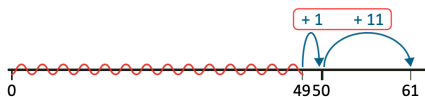


$$54 - 28:$$



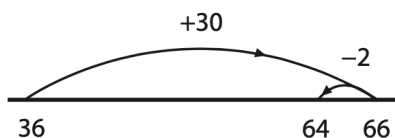
- **Counting on to find the difference** – to count on to find the difference when the numbers are close together.

$$61 - 49 = 12$$



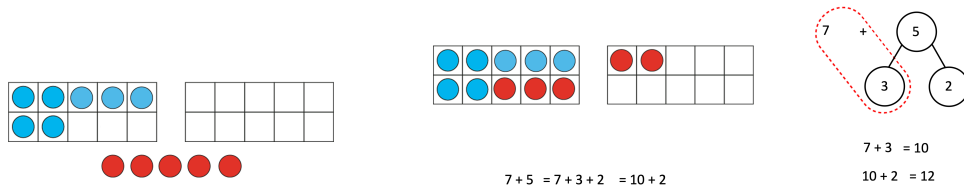
- **Compensation and adjusting** – to use rounding to add or subtract too much or little and adjust accordingly.

$$36 + 28 =$$

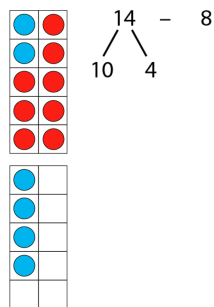




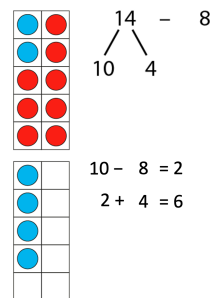
- **Bridging to next multiple of 10 or 100** - when adding or subtracting mentally, it is often useful to make use of the fact that one of the numbers is close to 10 or a multiple of 10 by partitioning another number to provide the difference  
 $7 + 5 =$



$$14 - 8$$



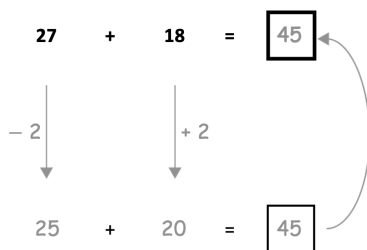
$$14 - 8 = 6$$



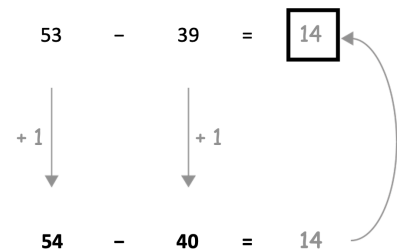
- **Redistribution** – to adjust parts of the addition and subtraction facts to make the calculation easier.

**Equal Sum:** the sum remains equal when we rebalance the addends in an addition calculation.

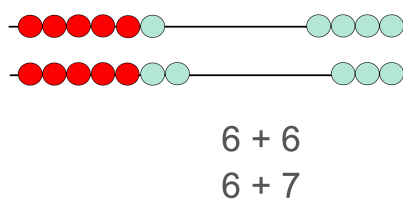
$$27 + 18 = 25 + 20 = 45$$



**Same Difference:** adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.



- **Using near doubles or halves** – using double facts and adjusting by adding or subtracting 1 to find near doubles.



# Key multiplication and division strategies

- 1. Knowing multiplication and division facts to  $12 \times 12$  – Times Table Challenge (see Factual Fluency)** - Fluent recall of multiplication and division facts relies on regular opportunities for practice. Generally, frequent short sessions are more effective than longer, less frequent sessions.

1 × 1	1 × 2	1 × 3	1 × 4	1 × 5	1 × 6	1 × 7	1 × 8	1 × 9	1 × 10	1 × 11	1 × 12
2 × 1	2 × 2	2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 8	2 × 9	2 × 10	2 × 11	2 × 12
3 × 1	3 × 2	3 × 3	3 × 4	3 × 5	3 × 6	3 × 7	3 × 8	3 × 9	3 × 10	3 × 11	3 × 12
4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 1	5 × 2	5 × 3	5 × 4	5 × 5	5 × 6	5 × 7	5 × 8	5 × 9	5 × 10	5 × 11	5 × 12
6 × 1	6 × 2	6 × 3	6 × 4	6 × 5	6 × 6	6 × 7	6 × 8	6 × 9	6 × 10	6 × 11	6 × 12
7 × 1	7 × 2	7 × 3	7 × 4	7 × 5	7 × 6	7 × 7	7 × 8	7 × 9	7 × 10	7 × 11	7 × 12
8 × 1	8 × 2	8 × 3	8 × 4	8 × 5	8 × 6	8 × 7	8 × 8	8 × 9	8 × 10	8 × 11	8 × 12
9 × 1	9 × 2	9 × 3	9 × 4	9 × 5	9 × 6	9 × 7	9 × 8	9 × 9	9 × 10	9 × 11	9 × 12
10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12

- 2. Doubling and halving** - The ability to double numbers is useful for multiplication. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations. Sometimes it can be helpful to halve one of the factors in a multiplication calculation and double the other, the product will stay the same.

- $14 \times 5 = 14 \times 10 \div 2$
- $12 \times 20 = 12 \times 2 \times 10$
- $60 \times 4 = 60 \times 2 \times 2$

- 3. Multiplying and dividing by multiples of 10** - Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers.



$$37 \times 10 =$$

- 4. Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers** - Once children are familiar with some multiplication facts, they can use these facts to work out others. One strategy that can be used is writing one of the numbers as the sum of two others about which more is known:  $6 \times 7 = 6 \times (2 + 5) = 6 \times 2 + 6 \times 5$ . Another strategy is making use of factors, so  $7 \times 6$  is seen as  $7 \times 3 \times 2$ .

## Distributive Law

Partition one of the numbers and use the distributive law of multiplication over addition.

$$6 \times 7 = 6 \times (5 + 2) = 6 \times 5 + 6 \times 2$$

Subtraction can be used similarly, so 'nine eights are ten eights minus one eight'.

## Factors

$7 \times 6$  is seen as  $7 \times 3 \times 2$ .

## Partitioning

$26 \times 3$  can be worked out by partitioning 26 into  $20 + 6$ , multiplying each part by 3, then recombining.

$$\begin{array}{r} 13 \times 12 \\ \swarrow \quad \searrow \\ 3 \quad 4 \\ 13 \times 3 \times 4 \end{array}$$

$$\begin{array}{r} 13 \times 12 \\ \swarrow \quad \searrow \quad \searrow \\ 3 \quad 2 \quad 2 \\ 13 \times 3 \times 2 \times 2 \end{array}$$

$$\begin{array}{r} 13 \times 12 \\ \swarrow \quad \searrow \\ 2 \quad 6 \\ 13 \times 2 \times 6 \end{array}$$

$$\begin{array}{r} 13 \times 12 \\ \swarrow \quad \searrow \quad \searrow \\ 2 \quad 2 \quad 3 \\ 13 \times 2 \times 2 \times 3 \end{array}$$

- 5. Finding fractions, decimals and percentages** - Children need an understanding of how fractions, decimals and percentages relate to each other, e.g. if they know that  $\frac{1}{2}$ , 0.5 and 50% are all ways of representing the same part of a whole, then the calculations

$$\begin{array}{l} \frac{1}{2} \times 40 \\ 40 \times 0.5 \\ 50\% \text{ of } £40 \end{array}$$

can be seen as different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.

# Mental calculation expectations for end of each phase

## End of Key Stage One

<p><b>Partitioning and Bridging</b></p> <p>5 + 6   7 + 4   9 + 7   7 + 6   8 + 7   7 + 5  2 + 18   4 + 18   8 + 19   47 + 6   68 + 7   9 + 87  13 - 8   27 - 8   53 - 6   68 - <math>\square</math> = 7   73 + <math>\square</math> = 89</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>8 + 6 = <math>\square</math>   5 + 7 = <math>\square</math>   12 - 7 = <math>\square</math>   46 + 7 = <math>\square</math>  8 + 5 + 4 = <math>\square</math>   55 + 17 = <math>\square</math>   71 - 14 = <math>\square</math>  86 - 21 = <math>\square</math>   65 + <math>\square</math> = 93</p> <p><b>'Think Addition' for subtraction</b></p> <p>8 - 5   9 - 6   6 - 2   80 - 50   19 - 6   60 - 20</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>10 - <math>\square</math> = 2   <math>\square</math> + 5 = 9   12 - 7 = <math>\square</math>   19 - 9 = <math>\square</math>  17 - 6 = <math>\square</math>   39 - 8 = <math>\square</math>   50 - <math>\square</math> = 20   56 - <math>\square</math> = 51</p> <p><b>Reordering and finding complements</b></p> <p>5 + 4 + 5   2 + 3 + 8   2 + 4 + 6   6 + 3 + 7  36 + 5 + 4   54 + 26</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>4 + 5 + 6 = <math>\square</math>   36 + 24 = <math>\square</math>   69 + 11 = <math>\square</math></p>	<p><b>Compensation</b></p> <p>2 + 9   12 + 9   9 + 72   2 + 19   19 + 42   42 + 39  5 + 8   15 + 8   65 + 8   18 + 5   55 + 18   48 + 35  12 - 9   22 - 9   52 - 9   52 - 19   92 - 19   92 - 39  12 - 8   22 - 8   52 - 8   52 - 18   92 - 18   92 - 48  48 + <math>\square</math> = 92   8 + <math>\square</math> = 52</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>8 + 6 = <math>\square</math>   52 + 7 = <math>\square</math>   28 + <math>\square</math> = 35   69 + 11 = <math>\square</math>  55 + 17 = <math>\square</math>   39 - 8 = <math>\square</math>   43 + 38 = <math>\square</math>   70 - 18 = <math>\square</math></p> <p><b>Redistribution - Equal sum</b></p> <p>12 + 9   9 + 72   24 + 19   15 + 42   44 + 37  5 + 8   15 + 8   65 + 7   18 + 6   55 + 15  48 + 35</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>8 + 6 = <math>\square</math>   4 + 5 + 6 = <math>\square</math>   69 + 11 = <math>\square</math>  55 + 17 = <math>\square</math>   36 + 24 = <math>\square</math>   43 + 38 = <math>\square</math>  8 + 5 + 4 = <math>\square</math></p> <p><b>Redistribution - Same difference</b></p> <p>32 - 7   25 - 8   55 - 7   55 - 17   92 - 19  97 - 43   48 + <math>\square</math> = 92   8 + <math>\square</math> = 55</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>12 - 7 = <math>\square</math>   28 + <math>\square</math> = 35   71 - 14 = <math>\square</math>  39 - 8 = <math>\square</math>   86 - 21 = <math>\square</math>   70 - 18 = <math>\square</math>  65 + <math>\square</math> = 93</p> <p><b>Reordering</b></p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>8 + 5 + 4 = <math>\square</math></p>	<p><b>Double and near double facts</b></p> <p>3 + 3   30 + 30   32 + 32   3 + 4   30 + 40  6 - 3   60 - 30   64 - 32   7 - 3   70 - 40  6 - <math>\square</math> = 3   60 - <math>\square</math> = 30   64 - <math>\square</math> = <math>\square</math>  <math>\square</math> - 3 = 64</p> <p>Find two ways of solving this: 70 - <math>\square</math> = <math>\square</math>   0 = <math>\square</math>   0</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>12 - 7 = <math>\square</math>   <math>\square</math> + 5 = 9   50 - <math>\square</math> = 20</p> <p><b>Make links to doubling and halving</b></p> <p>3 + 3 = 2 x 3   2 x 30   2 x 3 + 1  6 + 2   60 + 2</p> <p>Ensure that pupils can halve odd multiples of ten</p> <p>50 + 2 =</p> <p><i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i></p> <p>3 x 2 = <math>\square</math>   2 x 0 = <math>\square</math>  8 + 2 = <math>\square</math>   <math>\frac{1}{2}</math> of 16 = <math>\square</math>   <math>\frac{1}{2}</math> of 30 = <math>\square</math>  12 + 2 = <math>\square</math></p>
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## End of Lower Key Stage Two

<p><b>Partitioning and Bridging for addition</b></p> <p><b>Think 10</b></p> <p>37 + 45   68 + 23   29 + 75  76 + 27   55 + 16   42 + 38  537 + 8   727 + 5   213 + 18   146 + 37  36 - <math>\square</math> = 29   56 - 2 <math>\square</math> = 33   <math>\square</math> - 7 = 45 = 32</p> <p><b>Think 100</b></p> <p>290 + 13   370 + 50   580 + 73  270 + 51   67 + 350   860 + 69  86 + 770   680 + 63</p> <p><b>Think 1000</b></p> <p>4900 + 500   4800 + 260   6900 + 430  3200 + 910   230 + 7900   570 + 8500  3700 + 370   3622 + 500</p> <p><b>Think 1</b></p> <p>2.7 + 1.4   <math>2\frac{8}{10} + \frac{3}{10} =</math>   6.5 + 5.6  <math>1\frac{7}{8} + 1\frac{5}{8}</math></p> <p><b>'Partitioning and Bridging' for subtraction</b></p> <p><b>Think 10</b></p> <p>97 - 8   74 - 7   53 - 5   63 - 37  77 - 32   84 - 26   57 - 28   256 - 37  25 + <math>\square</math> = 85   163 + <math>\square</math> = 363   426 + 2 <math>\square</math> = 668</p> <p><b>Think 100</b></p> <p>230 - 70   660 - 82   420 - 77   950 - 147</p>	<p><b>Re-ordering and finding complements</b></p> <p><b>Complements to 10</b></p> <p>8 + 6 + 2 + 3 + 4   3 + 5 + 7 + 5 + 4  1 + 4 + 6 + 7 + 9   30 + 50 + 70  25 + 50 + 5   75 + 40 + 20 + 25</p> <p><b>Complements to 100</b></p> <p>400 + 547 + 600   700 + 240 + 300  750 + 400 + 250</p> <p><b>Complements to 1</b></p> <p>2.7 + 4 + 1.3   4.6 + 5 + 2.4   8.2 + 3 + 5.8</p> <p><b>Compensation</b></p> <p>42 + 29   45 + 27   24 + 47   28 + 65   68 + 27  232 + 49   856 + 17   48 + 325   232 + 95  132 + 59   568 + 195   399 + 423   412 + 298  405 + 199   597 + 308  43 - 18   94 - 37   54 - 29   77 - 9   82 - 23  483 - 99   256 - 98   398 - 74   597 - 63  401 - 97   736 - 301   613 - 299   743 - 397  298 - 156   799 - 403</p> <p><b>Redistribution - Equal sum</b></p> <p>45 + 27   26 + 39   78 + 18   65 + 27  73 + 39   84 + 47   42 + 97   116 + 35  368 + 123   404 + 198   356 + 427   528 + 298  3.7 + 1.9   7.6 + 4.7   1.9 + 5.8</p> <p><b>Redistribution - Same difference</b></p> <p>75 - 28   56 - 29   78 - 38   55 - 27  83 - 21   75 - 12   95 - 42   67 - 51  912 - 797   837 - 498   711 - 467   628 - 198  482 - 302   729 - 404   548 - 202   637 - 203  6.4 - 3.9   6.6 - 3.2   7.7 - 4.8   <math>1\frac{2}{7} - \frac{5}{7}</math></p> <p><b>Counting on to subtract</b></p> <p>315 - 298   412 - 396   917 - 898   611 - 598</p>	<p><b>Think multiplication</b></p> <p>85 + 5   72 + 4   99 + 6   240 + 12  660 + 3   210 + 7   540 + 9   500 + 4  <math>\square \div 3 = 8</math>   <math>3\square \div 5 = 6</math></p> <p><b>X and + 10, 100 and 1000</b></p> <p>4 x 30   9 x 30   70 x 70   60 x 50  300 x 4   800 x 7   9 x 800   6 x 400  3 x 2000   4000 x 6   8 x 7000   9 x 8000  500 + 10   400 + 5   600 + 2   240 + 4  120 + <math>\square</math> = 12   365cm = <math>\square</math> m   750mm = <math>\square</math> cm</p> <p><b>Double and near double facts</b></p> <p>7 x 20   3 x 38   9 x 200   11 x 4  16 x 20   18 x 2000  80 + 4   160 + 4   1600 + 4   2400 + 4</p> <p><b>Think 5 / Think 10 for multiplication</b></p> <p>28 x 5   16 x 8   23 x 9   92 x 8   52 x 4  13 x 21   34 x 19   123 x 4   214 x 6   9 x 234  11 x 314   21 x 400   400 x 38</p>
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## End of Upper Key Stage Two

<p><b>Place Value</b></p> <p>937 + 100      1969 + 100      546 - 40  1.7 + 0.05      40 000 - 500  246 ÷ 1      100 x 217      0.4 ÷ 10  1.68 x 100      100 x 100</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  435 - 30    979 + 100    3.005 + 6.12    2.15 + 0.05  100 x 412    0.9 ÷ 10    1.28 x 100    50,000 - 500  10 x 100</p> <p>Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?  Circle two numbers that added together make 0.25  0.05    0.23    0.2    0.5</p> <p>Circle two numbers that multiply together to equal 1 million  200    2,000    5,000    50,000</p> <p>Write the number that is 5 less than 10 million  Write the number that is one hundred thousand less than six million  Round 124,531 to the nearest 10,000, 1,000, 100</p> <p><b>Partitioning and Bridging</b></p> <p>58 + 6      5 + 47      630 + 73      680 + 78  560 + 89      8900 + 230  74 - 7      97 - 8      320 - 50      2300 - 600  3400 - 1700</p> <p>5 - 2.65      8.1 - 2.75      <math>1\frac{2}{5} + \frac{3}{10} =</math>      <math>1\frac{3}{10} - \frac{2}{5} =</math>  £3367.40 - £1021.23</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  4 - 1.15    <math>1\frac{4}{5} + \frac{3}{10}</math>    <math>1\frac{1}{4} + \frac{1}{3}</math>    <math>1\frac{1}{5} - \frac{1}{4}</math>    <math>\frac{3}{4} + \frac{7}{8} =</math>  5,756 + 8,643    936 + 285</p>	<p><b>Compensation</b></p> <p>56 + 8      72 + 9      56 - 8      72 - 9  371 + 18      255 + 49      304 + 299  673 - 99      854 - 398      3720 - 996  0.71 + 0.09      0.56 + 0.08      0.34 - 0.09  £1.17 + £0.39      £8.89 - £4.99</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  468 - 9      472 - 9      15.98 + 26.314  12 - 6.01      15.4 - 8.88</p> <p><b>Redistribution - Equal sum</b></p> <p>56 + 8      72 + 9      371 + 18      255 + 49  304 + 267  £37.67 + £3.85      563 + 397      890,488 + 4,890  229,899 + 31,321</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  89,994 + 7,643      936 + 285      89,994 + 7,643</p> <p><b>Redistribution - Same difference</b></p> <p>85 - 18      42 - 17      88 - 43      437 - 103  819 - 504      532,525 - 9897  £122.56 - £87.99      9.1 - 6.7      15.3 - 5.7</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  468 - 9      472 - 9      122,456 - 11,999  4 - 1.15      12 - 6.01  15.4 - 8.88      234,897 - 45,996</p>	<p><b>Think Partition for x and ÷</b></p> <p>32 x 4      29 x 2      122 x 4      4.6 x 2  75 x 3      8.3 x 6      39 x 7  3.3 x 7      5 x 49      4 x 198      96 x 0.3</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  15 x 6.1    24 x 3    1.52 x 6    7,505 ÷ 5  17 x 1½</p> <p><b>Make links to doubling and halving</b></p> <p>50 x 28    86 x 50    500 x 70    18 x 2.5  86 x 2.5    160 x 35    500 x 88    1.5 x 6.6  0.5 x 120    4.5 x 2.2    15% x 346    75% x 220</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  15% x 440    <math>\frac{2}{5} \times 140</math>    24 x 3  20% of 1500    95% of 240</p> <p><b>Multiplying and dividing fractions</b></p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  <math>\frac{3}{5} \div 3</math>    <math>\frac{2}{5} \div 2</math>    <math>\frac{3}{4} \div 2</math>    <math>\frac{2}{5} \times 140</math>    <math>\frac{1}{4} \times \frac{1}{8}</math></p>
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<p><b>Re-ordering and finding complements</b></p> <p>11 + 59      33 + 57      14 + 90 + 86  290 + 310      1.15 + 2.55      0.8 + 0.26</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  1,034 + 586    2.15 + 0.05  Circle two numbers that added together make 0.25  0.05    0.23    0.2    0.5</p>	<p><b>x and ÷ by powers of 10</b></p> <p>10 x 53    87 x 10    1000 x 14    100 x 8.3  100 x 0.41  30 x 3    7 x 0.3    30 x 30    30 x 70  567 ÷ 100    36 ÷ 10    0.5 ÷ 10    280 ÷ 4</p> <p>5600 ÷ 80    30 = □ ÷ 12    270 ÷ 9 = □ ÷ 0.9  7 x 0.001    1.8 ÷ 0.1    3.25 ÷ 0.00001</p> <p>Circle two numbers that multiply together to equal 10 million  200    2,000    5,000    50,000</p> <p><i>Examples from 2016 KS2 and Sample Papers</i>  1440 ÷ 12    630 ÷ 9    1,320 ÷ 12  0.9 ÷ 10  20% of 1,800    20% of 1500    7,505 ÷ 5  95% of 240  100 x 412    0.9 ÷ 10    1.28 x 100  50,000 - 500    10 x 100</p> <p>Circle two numbers that multiply together to equal 1 million  200    2,000    5,000    50,000</p>	
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## End of Key Stage Three

Consolidation of the end of KS2 expectations to support fluency aims being accurate, flexible in your approach to calculation and choosing the most efficient methods.

## Additional Guidance Key Stage One

- **Counting forwards and backwards** – in a variety of interval steps

$4 + 5$	count on in ones from 4 (or in ones from 5)
$8 - 3$	count back in ones from 8
$10 + 7$	count on in ones from 10 (or use place value)
$13 + 5$	count on in ones from 13
$17 - 3$	count back in ones from 17
$18 - 6$	count back in twos
$23 + 5$	count on in ones from 23
$57 - 3$	count back in ones from 57
$60 + 5$	count on in ones from 60 (or use place value)
$80 - 7$	count back in ones from 80 (or use knowledge of number facts to 10 and place value)
$27 + 60$	count on in tens from 27
$72 - 50$	count back in tens from 72

- **Reordering** – know when and how to reorder to make calculations easier

$$2 + 7 = 7 + 2$$

$$5 + 13 = 13 + 5$$

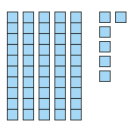
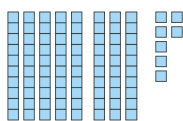
- **Finding complements** – identifying pairs or trios of 1, 10, 20 (like reordering)

$2 + 7$	$7 + 2$
$5 + 13$	$13 + 5$
$10 + 2 + 10$	$10 + 10 + 2$
$5 + 34$	$34 + 5$
$5 + 7 + 5$	$5 + 5 + 7$

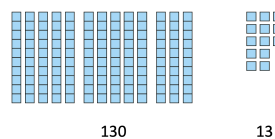
- **Partitioning (regrouping)** – the ability to break numbers up and recombine them flexibly. It is important that children are aware that numbers can be partitioned – both along the place value boundaries (canonically) and in other ways (non-canonically).

$$\begin{array}{r} 87 \\ / \quad \backslash \\ 80 \quad 7 \end{array} + \begin{array}{r} 56 \\ / \quad \backslash \\ 50 \quad 6 \end{array}$$

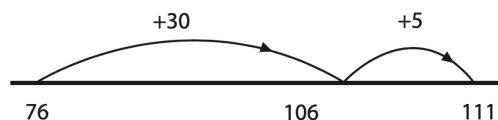
$$\begin{array}{r} 87 \\ / \quad \backslash \\ 80 \quad 7 \end{array} + \begin{array}{r} 56 \\ / \quad \backslash \\ 50 \quad 6 \end{array} = 130 + 13$$



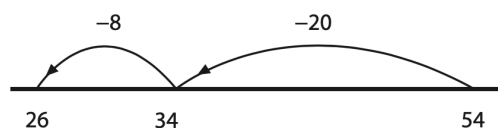
then



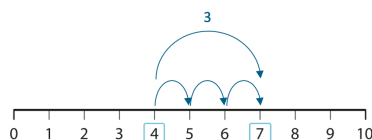
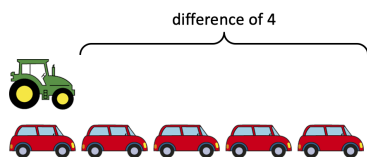
$$76 + 35:$$



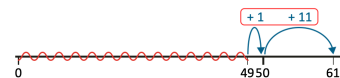
$$54 - 28:$$



- **Counting on to find the difference** – to count on to find the difference when the numbers are close together.



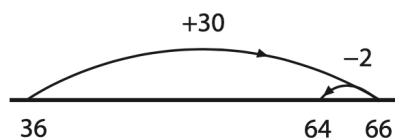
$$61 - 49 = 12$$



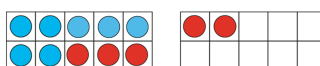
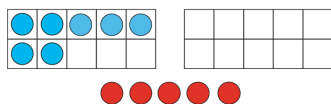
- **Compensation and adjusting** – to use rounding to add or subtract too much or little and adjust accordingly.

$34 + 9$ $34 + 19$ $34 + 29$ and so on	$34 + 10 - 1$ $34 + 20 - 1$ $34 + 30 - 1$ and so on
$34 + 11$ $34 + 21$ $34 + 31$ and so on	$34 + 10 + 1$ $34 + 20 + 1$ $34 + 30 + 1$ and so on
$70 - 9$	$70 - 10 + 1$

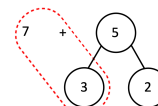
$$36 + 28 =$$



- **Bridging to next multiple of 10 or 100** - when adding or subtracting mentally, it is often useful to make use of the fact that one of the numbers is close to 10 or a multiple of 10 by partitioning another number to provide the difference



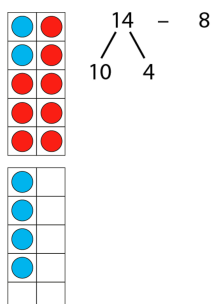
$$7 + 5 = 7 + 3 + 2 = 10 + 2$$



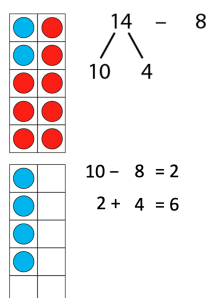
$$7 + 3 = 10$$

$$10 + 2 = 12$$

$$14 - 8$$



$$14 - 8 = 6$$



$$10 - 8 = 2$$

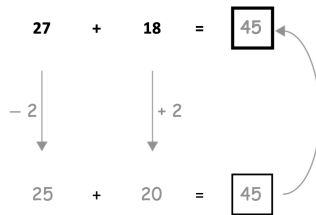
$$2 + 4 = 6$$



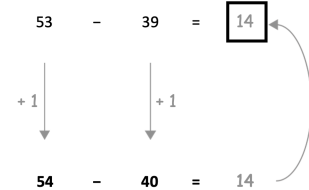
- **Redistribution** – to adjust parts of the addition and subtraction facts to make the calculation easier.

- **Equal Sum:** the sum remains equal when we rebalance the addends in an addition calculation.

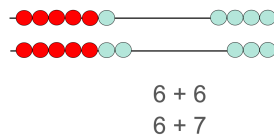
$$27 + 18 = 25 + 20 = 45$$



- **Same Difference:** adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.



- **Using near doubles or halves** – using double facts and adjusting by adding or subtracting 1 to find near doubles.



$$6 + 6$$

$$6 + 7$$

## Key multiplication and division strategies

<b>Year 1</b> <ul style="list-style-type: none"> <li>doubles of all numbers to 10, e.g. double 6</li> <li>odd and even numbers to 20</li> </ul>	<ul style="list-style-type: none"> <li>count on from and back to zero in ones, twos, fives or tens</li> </ul>	<ul style="list-style-type: none"> <li>use patterns of last digits, e.g. 0 and 5 when counting in fives</li> </ul>
<b>Year 2</b> <ul style="list-style-type: none"> <li>doubles of all numbers to 20, e.g. double 13, and corresponding halves</li> <li>doubles of multiples of 10 to 50, e.g. double 40, and corresponding halves</li> <li>multiplication facts for the 2, 5 and 10 times-tables, and corresponding division facts</li> <li>odd and even numbers to 100</li> </ul>	<ul style="list-style-type: none"> <li>double any multiple of 5 up to 50, e.g. double 35</li> <li>halve any multiple of 10 up to 100, e.g. halve 90</li> <li>find half of even numbers to 40</li> <li>find the total number of objects when they are organised into groups of 2, 5 or 10</li> </ul>	<ul style="list-style-type: none"> <li>partition: double the tens and ones separately, then recombine</li> <li>use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two</li> <li>use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five</li> </ul>

## End of Key Stage One Expectations

<b>Partitioning and Bridging</b> $5 + 6$ $7 + 4$ $9 + 7$ $7 + 6$ $8 + 7$ $7 + 5$ $2 + 18$ $4 + 18$ $8 + 19$ $47 + 6$ $68 + 7$ $9 + 87$ $13 - 8$ $27 - 8$ $53 - 6$ $68 - \square = 7$ $73 + \square = 89$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $8 + 6 = \square$ $5 + 7 = \square$ $12 - 7 = \square$ $46 + 7 = \square$ $8 + 5 + 4 = \square$ $55 + 17 = \square$ $71 - 14 = \square$ $86 - 21 = \square$ $65 + \square = 93$  <b>'Think Addition' for subtraction</b> $8 - 5$ $9 - 6$ $6 - 2$ $80 - 50$ $19 - 6$ $60 - 20$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $10 - \square = 2$ $\square + 5 = 9$ $12 - 7 = \square$ $19 - 9 = \square$ $17 - 6 = \square$ $39 - 8 = \square$ $50 - \square = 20$ $56 - \square = 51$  <b>Reordering and finding complements</b> $5 + 4 + 5$ $2 + 3 + 8$ $2 + 4 + 6$ $6 + 3 + 7$ $36 + 5 + 4$ $54 + 26$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $4 + 5 + 6 = \square$ $36 + 24 = \square$ $69 + 11 = \square$	<b>Compensation</b> $2 + 9$ $12 + 9$ $9 + 72$ $2 + 19$ $19 + 42$ $42 + 39$ $5 + 8$ $15 + 8$ $65 + 8$ $18 + 5$ $55 + 18$ $48 + 35$ $12 - 9$ $22 - 9$ $52 - 9$ $52 - 19$ $92 - 19$ $92 - 39$ $12 - 8$ $22 - 8$ $52 - 8$ $52 - 18$ $92 - 18$ $92 - 48$ $48 + \square = 92$ $8 + \square = 52$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $8 + 6 = \square$ $52 + 7 = \square$ $28 + \square = 35$ $69 + 11 = \square$ $55 + 17 = \square$ $39 - 8 = \square$ $43 + 38 = \square$ $70 - 18 = \square$  <b>Redistribution - Equal sum</b> $12 + 9$ $9 + 72$ $24 + 19$ $15 + 42$ $44 + 37$ $5 + 8$ $15 + 8$ $65 + 7$ $18 + 6$ $55 + 15$ $48 + 35$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $8 + 6 = \square$ $4 + 5 + 6 = \square$ $69 + 11 = \square$ $55 + 17 = \square$ $36 + 24 = \square$ $43 + 38 = \square$ $8 + 5 + 4 = \square$  <b>Redistribution - Same difference</b> $32 - 7$ $25 - 8$ $55 - 7$ $55 - 17$ $92 - 19$ $97 - 43$ $48 + \square = 92$ $8 + \square = 55$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $12 - 7 = \square$ $28 + \square = 35$ $71 - 14 = \square$ $39 - 8 = \square$ $86 - 21 = \square$ $70 - 18 = \square$ $65 + \square = 93$  <b>Reordering</b> <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $8 + 5 + 4 = \square$	<b>Double and near double facts</b> $3 + 3$ $30 + 30$ $32 + 32$ $3 + 4$ $30 + 40$ $6 - 3$ $60 - 30$ $64 - 32$ $7 - 3$ $70 - 40$ $6 - \square = 3$ $60 - \square = 30$ $64 - \square =$ $\square - 3 = 64$  Find two ways of solving this: $70 - \square = \square$ $0 = \square$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $12 - 7 = \square$ $\square + 5 = 9$ $50 - \square = 20$  <b>Make links to doubling and halving</b> $3 + 3 = 2 \times 3$ $2 \times 30$ $2 \times 3 + 1$ $6 + 2$ $60 + 2$  Ensure that pupils can halve odd multiples of ten $50 \div 2 =$  <i>Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper</i> $3 \times 2 = \square$ $2 \times 0 = \square$ $8 \div 2 = \square$ $\frac{1}{2}$ of 16 = $\square$ $\frac{1}{2}$ of 30 = $\square$ $12 \div 2 = \square$
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## Additional Guidance Lower Key Stage Two

- **Counting forwards and backwards** – in a variety of interval steps

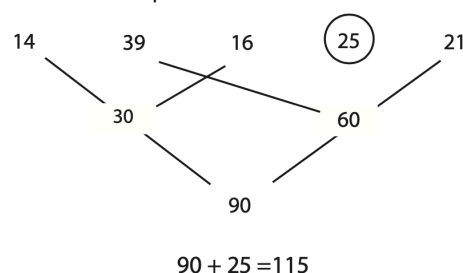
$50 + 38$	count on in tens then ones from 50	$73 - 68$	count up from 68, counting 2 to 70 then 3 to 73
$90 - 27$	count back in tens then ones from 90	$47 + 58$	count on 50 from 47, then 3 to 100, then 5 to 105
$34 + 65$	count on in tens then ones from 34	$124 - 47$	count back 40 from 124, then 4 to 80, then 3 to 77
$87 - 23$	count back in tens then ones from 87	$570 + 300$	count on in hundreds from 570
$35 + 15$	count on in steps of 5 from 35	$960 - 500$	count back in hundreds from 960

- **Reordering** – know when and how to reorder to make calculations easier
- **Finding complements** – identifying pairs or trios of 1, 10, 20 and 100...(like reordering)

$23 + 54$	$54 + 23$
$12 - 7 - 2$	$12 - 2 - 7$
$13 + 21 + 13$	$13 + 13 + 21$ (using double 13)
$6 + 13 + 4 + 3$	$6 + 4 + 13 + 3$
$17 + 9 - 7$	$17 - 7 + 9$
$28 + 75$	$75 + 28$ (thinking of 28 as 25 + 3)

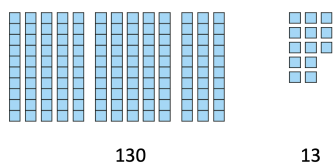
$$14 + 39 + 16 + 25 + 21$$

it is sensible to pair numbers:

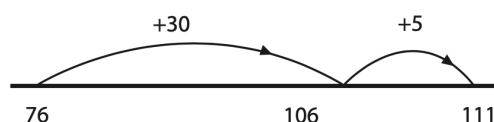


- **Partitioning (regrouping)** – the ability to break numbers up and recombine them flexibly. It is important that children are aware that numbers can be partitioned – both along the place value boundaries (canonically) and in other ways (non-canonically).

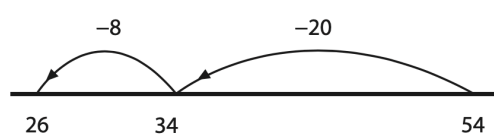
$$\begin{array}{r} 87 \\ 80 \end{array} + \begin{array}{r} 56 \\ 50 \end{array} = 130 + 13$$



$$76 + 35:$$



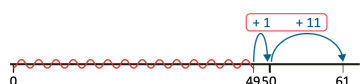
$$54 - 28:$$



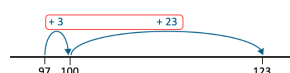
$23 + 45$	$40 + 5 + 20 + 3 = 40 + 20 + 5 + 3$
$68 - 32$	$60 + 8 - 30 - 2 = 60 - 30 + 8 - 2$
$55 + 37$	$55 + 30 + 7 = 85 + 7$
$365 - 40$	$300 + 60 + 5 - 40 = 300 + 60 - 40 + 5$

- **Counting on to find the difference** – to count on to find the difference when the numbers are close together.

$$61 - 49 = 12$$



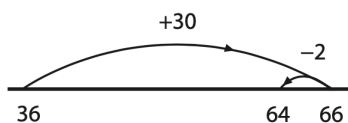
$$123 - 97 = 26$$



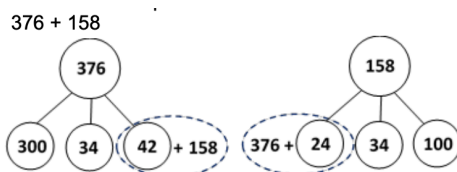
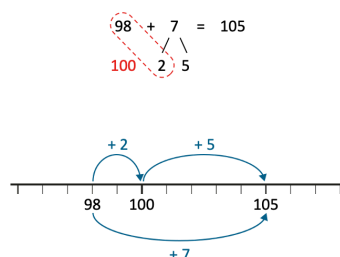
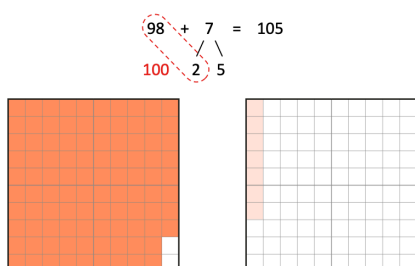
- **Compensation and adjusting** – to use rounding to add or subtract too much or little and adjust accordingly.

$53 + 12$	$53 + 10 + 2$
$53 - 12$	$53 - 10 - 2$
$53 + 18$	$53 + 20 - 2$
$84 - 18$	$84 - 20 + 2$
$38 + 68$	$38 + 70 - 2$
$95 - 78$	$95 - 80 + 2$
$58 + 32$	$58 + 30 + 2$
$64 - 32$	$64 - 30 - 2$

$$36 + 28 =$$



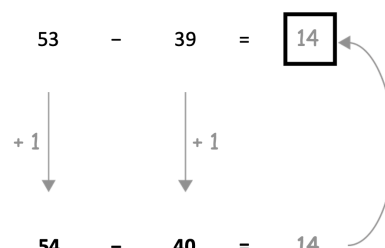
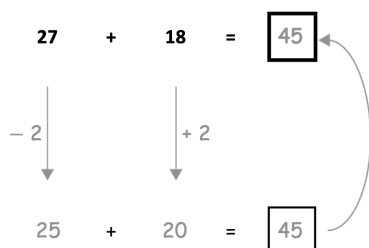
- **Bridging to next multiple of 10 or 100** - when adding or subtracting mentally, it is often useful to make use of the fact that one of the numbers is close to 10 or a multiple of 10 by partitioning another number to provide the difference



$49 + 32$	$49 + 1 + 31$
$90 - 27$	$27 + 3 + 60$
$57 + 34$ or $92 - 25$	$57 + 3 + 31$ or $92 - 2 - 20 - 3$
$84 - 35$	$35 + 5 + 40 + 4$

- **Redistribution** – to adjust parts of the addition and subtraction facts to make the calculation easier.
- **Equal Sum:** the sum remains equal when we rebalance the addends in an addition calculation.
- **Same Difference:** adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.

$$27 + 18 = 25 + 20 = 45$$



- **Using near doubles or halves** – using double facts and adjusting by adding or subtracting 1 to find near doubles.

18 + 16	is double 18 and subtract 2 or double 16 and add 2
60 + 70	is double 60 and add 10 or double 70 and subtract 10
76 + 75	is double 76 and subtract 1 or double 75 and add 1

## Key multiplication and division strategies

- **Knowing multiplication and division facts to  $12 \times 12$  – Times Table Challenge (see Factual Fluency)**
- **Doubling and halving** - The ability to double numbers is useful for multiplication. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations. Sometimes it can be helpful to halve one of the factors in a multiplication calculation and double the other, the product will stay the same.

Double multiples of 10 to 100, e.g. double 90, and corresponding halves Double multiples of 5 to 100 and find the corresponding halves, e.g. double 85, halve 170
Double any two-digit number and find the corresponding halves, e.g. double 47, half of 94
Double multiples of 10 and 100 and find the corresponding halves, e.g. double 800, double 340, half of 1600, half of 680

- **Multiplying and dividing by multiples of 10** - Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers.

			37
Thousands	Hundreds	Tens	Ones

			370
Thousands	Hundreds	Tens	Ones

$$\square = 3 \times 40$$



3 groups of 4 tens is equal to 12 tens.  
12 tens is 120.  
So  $120 = 3 \times 40$ .

$$37 \times 10 =$$

Multiply one-digit and two-digit numbers by 10 or 100, e.g. $7 \times 100$ , $46 \times 10$ , $54 \times 100$
Change pounds to pence, e.g. £6 to 600 pence, £1.50 to 150 pence
Multiply numbers to 1000 by 10 and then 100, e.g. $325 \times 10$ , $42 \times 100$
Divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. $120 \div 10$ , $600 \div 100$ , $850 \div 10$
Multiply a multiple of 10 to 100 by a single-digit number, e.g. $60 \times 3$ , $50 \times 7$
Change hours to minutes; convert between units involving multiples of 10 and 100, e.g. centimetres and millimetres, centilitres and millilitres, and convert between pounds and pence, metres and centimetres, e.g. 599 pence to £5.99, 2.5m to 250cm

➤ **Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers**

Find one quarter by halving one half

Multiply numbers to 20 by a single-digit number, e.g.  $17 \times 3$

➤ **Finding fractions, decimals and percentages** - Children need an understanding of how fractions, decimals and percentages relate to each other, e.g. if they know that  $\frac{1}{2}$ , 0.5 and 50% are all ways of representing the same part of a whole, then the calculations

Find half of any multiple of 10 up to 200, e.g. halve 170

Find  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  and  $\frac{1}{10}$  of numbers in the 2, 3, 4, 5 and 10 times tables

Find half of any even number to 200

Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g.  $\frac{3}{8}$  of 24

Recall fraction and decimal equivalents for one-half, quarters, tenths and hundredths, e.g. recall the equivalence of 0.3 and  $\frac{3}{10}$ , and 0.03 and  $\frac{3}{100}$

**Partitioning and Bridging for addition**

**Think 10**

$37 + 45$        $68 + 23$        $29 + 75$   
 $76 + 27$        $55 + 16$        $42 + 38$   
 $537 + 8$        $727 + 5$        $213 + 18$        $146 + 37$   
 $36 - \square = 29$        $56 - 2\square = 33$        $\square7 - 45 = 32$

**Think 100**

$290 + 13$        $370 + 50$        $580 + 73$   
 $270 + 51$        $67 + 350$        $860 + 69$   
 $86 + 770$        $680 + 63$

**Think 1000**

$4900 + 500$        $4800 + 260$        $6900 + 430$   
 $3200 + 910$        $230 + 7900$        $570 + 8500$   
 $3700 + 370$        $3622 + 500$

**Think 1**

$2.7 + 1.4$        $2\frac{8}{10} + \frac{3}{10} =$        $6.5 + 5.6$   
 $1\frac{7}{8} + 1\frac{5}{8}$

**Partitioning and Bridging for subtraction**

**Think 10**

$97 - 8$        $74 - 7$        $53 - 5$        $63 - 37$   
 $77 - 32$        $84 - 26$        $57 - 28$        $256 - 37$   
 $25 + \square = 85$        $163 + \square = 363$        $426 + 2\square2 = 668$

**Think 100**

$230 - 70$        $660 - 82$        $420 - 77$        $950 - 147$

**Re-ordering and finding complements**

**Complements to 10**

$8 + 6 + 2 + 3 + 4$        $3 + 5 + 7 + 5 + 4$   
 $1 + 4 + 6 + 7 + 9$        $30 + 50 + 70$   
 $25 + 50 + 5$        $75 + 40 + 20 + 25$

**Complements to 100**

$400 + 547 + 600$        $700 + 240 + 300$   
 $750 + 400 + 250$

**Complements to 1**

$2.7 + 4 + 1.3$        $4.6 + 5 + 2.4$        $8.2 + 3 + 5.8$

**Compensation**

$42 + 29$        $45 + 27$        $24 + 47$        $28 + 65$        $68 + 27$   
 $232 + 49$        $856 + 17$        $48 + 325$        $232 + 95$   
 $132 + 59$        $568 + 195$        $399 + 423$        $412 + 298$   
 $405 + 199$        $597 + 308$   
 $43 - 18$        $94 - 37$        $54 - 29$        $77 - 9$        $82 - 23$   
 $483 - 99$        $256 - 98$        $398 - 74$        $597 - 63$   
 $401 - 97$        $736 - 301$        $613 - 299$        $743 - 397$   
 $298 - 156$        $799 - 403$

**Redistribution - Equal sum**

$45 + 27$        $26 + 39$        $78 + 18$        $65 + 27$   
 $73 + 39$        $84 + 47$        $42 + 97$        $116 + 35$   
 $368 + 123$        $404 + 198$        $356 + 427$        $528 + 298$   
 $3.7 + 1.9$        $7.6 + 4.7$        $1.9 + 5.8$

**Redistribution - Same difference**

$75 - 28$        $56 - 29$        $78 - 38$        $55 - 27$   
 $83 - 21$        $75 - 12$        $95 - 42$        $67 - 51$   
 $912 - 797$        $837 - 498$        $711 - 467$        $628 - 198$   
 $482 - 302$        $729 - 404$        $548 - 202$        $637 - 203$   
 $6.4 - 3.9$        $6.6 - 3.2$        $7.7 - 4.8$        $1\frac{2}{7} - \frac{5}{7}$

**Counting on to subtract**

$315 - 298$        $412 - 396$        $917 - 898$        $611 - 598$

**Think multiplication**

$85 \div 5$        $72 \div 4$        $99 \div 6$        $240 \div 12$   
 $660 \div 3$        $210 \div 7$        $540 \div 9$        $500 \div 4$   
 $\square \div 3 = 8$        $3\square \div 5 = 6$

**X and  $\div$  10, 100 and 1000**

$4 \times 30$        $9 \times 30$        $70 \times 70$        $60 \times 50$   
 $300 \times 4$        $800 \times 7$        $9 \times 800$        $6 \times 400$   
 $3 \times 2000$        $4000 \times 6$        $8 \times 7000$        $9 \times 8000$   
 $500 \div 10$        $400 \div 5$        $600 \div 2$        $240 \div 4$   
 $120 \div \square = 12$        $365\text{cm} = \square \text{m}$        $750\text{mm} = \square \text{cm}$

**Double and near double facts**

$7 \times 20$        $3 \times 38$        $9 \times 200$        $11 \times 4$   
 $16 \times 20$        $18 \times 2000$   
 $80 \div 4$        $160 \div 4$        $1600 \div 4$        $2400 \div 4$

**Think 5 / Think 10 for multiplication**

$28 \times 5$        $16 \times 8$        $23 \times 9$        $92 \times 8$        $52 \times 4$   
 $13 \times 21$        $34 \times 19$        $123 \times 4$        $214 \times 6$        $9 \times 234$   
 $11 \times 314$        $21 \times 400$        $400 \times 38$

## Additional Guidance Upper Key Stage Two

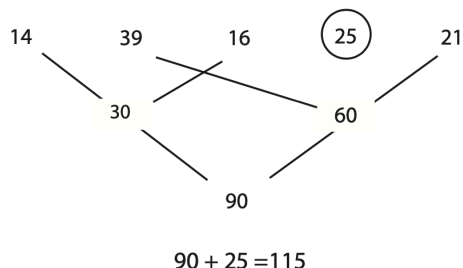
- **Counting forwards and backwards** – in a variety of interval steps

$3.2 + 0.6$	count on in tenths
$1.7 + 0.55$	count on in tenths and hundredths

- **Reordering** – know when and how to reorder to make calculations easier
- **Finding complements** – identifying pairs or trios of 1, 10, 20 and 100...(like reordering)

$$14 + 39 + 16 + 25 + 21$$

it is sensible to pair numbers:



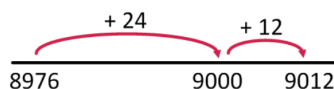
$12 + 17 + 8 + 3$	$12 + 8 + 17 + 3$
$25 + 36 + 75$	$25 + 75 + 36$
$58 + 47 - 38$	$58 - 38 + 47$
$200 + 567$	$567 + 200$
$1.7 + 2.8 + 0.3$	$1.7 + 0.3 + 2.8$
$3 + 8 + 7 + 6 + 2$	$3 + 7 + 8 + 2 + 6$
$34 + 27 + 46$	$34 + 46 + 27$
$180 + 650$	$650 + 180$ (thinking of 180 as 150 + 30)
$1.7 + 2.8 + 0.3$	$1.7 + 0.3 + 2.8$
$4.7 + 5.6 - 0.7$	$4.7 - 0.7 + 5.6 = 4 + 5.6$

- **Partitioning (regrouping)** – the ability to break numbers up and recombine them flexibly. It is important that children are aware that numbers can be partitioned – both along the place value boundaries (canonically) and in other ways (non-canonically).

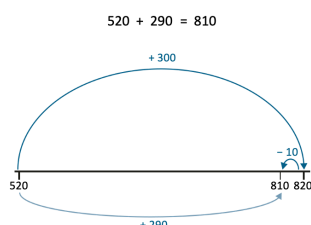
$43 + 28 + 51$	$40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1$
$5.6 + 3.7$	$5.6 + 3 + 0.7 = 8.6 + 0.7$
$4.7 - 3.5$	$4.7 - 3 - 0.5$
$540 + 280$	$540 + 200 + 80$
$276 - 153$	$276 - 100 - 50 - 3$

- **Counting on to find the difference** – to count on to find the difference when the numbers are close together.

$$9012 - 8976$$



- **Compensation and adjusting** – to use rounding to add or subtract too much or little and adjust accordingly.

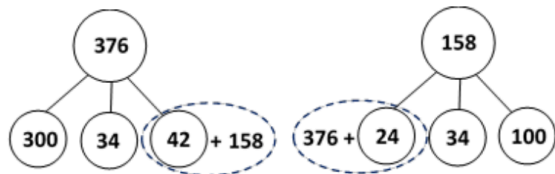


$138 + 69$	$138 + 70 - 1$
$405 - 399$	$405 - 400 + 1$
$2\frac{1}{2} + 1\frac{3}{4}$	$2\frac{1}{2} + 2 - \frac{1}{4}$
$5.7 + 3.9$	$5.7 + 4.0 - 0.1$
$6.8 - 4.9$	$6.8 - 5.0 + 0.1$



- **Bridging to next multiple of 1, 10, 100 or 1000** - when adding or subtracting mentally, it is often useful to make use of the fact that one of the numbers is close to 10 or a multiple of 10 by partitioning another number to provide the difference

$$376 + 158$$



$607 - 288$	$288 + 12 + 300 + 7$
$6070 - 4987$	$4987 + 13 + 1000 + 70$
$1.4 + 1.7$ or $5.6 - 3.7$	$1.4 + 0.6 + 1.1$ or $5.6 - 0.6 - 3 - 0.1$
$0.8 + 0.35$	$0.8 + 0.2 + 0.15$
$8.3 - 2.8$	$2.8 + 0.2 + 5.3$ or $8.3 - 2.3 - 0.5$

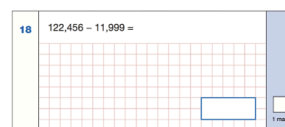
- **Redistribution** – to adjust parts of the addition and subtraction facts to make the calculation easier.

**Equal Sum:** the sum remains equal when we rebalance the addends in an addition calculation.



$$\begin{array}{rcl}
 89,994 & + & 7,643 = 97,637 \\
 \downarrow +6 & & \downarrow -6 \\
 90,000 & + & 7,637 = 97,637
 \end{array}$$

**Same Difference:** adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.



$$\begin{array}{rcl}
 122,456 & - & 11,999 = 110,457 \\
 \downarrow +1 & & \downarrow +1 \\
 122,457 & - & 12,000 = 110,457
 \end{array}$$

$$5.3 + 3.98 = 5.28 + 4.0 = 9.28$$

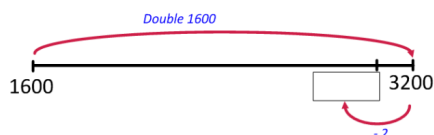
$$\begin{array}{rcl}
 5.3 & + & 3.98 = 9.28 \\
 \downarrow -0.02 & & \downarrow +0.02 \\
 5.28 & + & 4.0 = 9.28
 \end{array}$$

$$20,000 - 1,658$$

$$\begin{array}{r}
 \overset{1}{\cancel{2}} \overset{1}{\cancel{0}}, \overset{9}{\cancel{0}} \overset{9}{\cancel{0}} \overset{9}{\cancel{0}} \\
 - \quad 1,658 \\
 \hline
 18,342
 \end{array}$$

- **Using near doubles or halves** – using double facts and adjusting by adding or subtracting 1 to find near doubles.

$$1600 + 1598 = \text{double } 1600 - 2$$



$160 + 170$	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
$2.5 + 2.6$	is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

## Key multiplication and division strategies

- **Knowing multiplication and division facts to  $12 \times 12$  – Times Table Challenge (see Factual Fluency)**

$$\begin{array}{c}
 6 \div 2 = 3 \qquad 4 \times 3 = 12 \\
 6 \div 3 = 2 \qquad 20 \times 3 = 60 \\
 \frac{1}{2} (4 \times 3) = 6 \quad \text{---} \quad 2 \times 3 = 6 \quad \text{---} \quad 200 \times 3 = 600 \\
 2 \times 1\frac{1}{2} = 3 \qquad 40 \times 3 = 120 \\
 12 \times 3 = 10 \times 3 + 2 \times 3 = 30 + 6 = 36
 \end{array}$$

- **Doubling and halving** - The ability to double numbers is useful for multiplication. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations. Sometimes it can be helpful to halve one of the factors in a multiplication calculation and double the other, the product will stay the same.

Form equivalent calculations and use doubling and halving, e.g.
<ul style="list-style-type: none"> <li>• multiply by 4 by doubling twice, e.g. <math>16 \times 4 = 32 \times 2 = 64</math></li> <li>• multiply by 8 by doubling three times, e.g. <math>12 \times 8 = 24 \times 4 = 48 \times 2 = 96</math></li> <li>• divide by 4 by halving twice, e.g. <math>104 \div 4 = 52 \div 2 = 26</math></li> <li>• divide by 8 by halving three times, e.g. <math>104 \div 8 = 52 \div 4 = 26 \div 2 = 13</math></li> <li>• multiply by 5 by multiplying by 10 then halving, e.g. <math>18 \times 5 = 180 \div 2 = 90</math></li> <li>• multiply by 20 by doubling then multiplying by 10, e.g. <math>53 \times 20 = 106 \times 10 = 1060</math></li> </ul>
Multiply by 50 by multiplying by 100 and halving
Multiply by 25 by multiplying by 100 and halving twice
Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2
Form equivalent calculations and use doubling and halving, e.g.
<ul style="list-style-type: none"> <li>• divide by 25 by dividing by 100 then multiplying by 4 e.g. <math>460 \div 25 = 4.6 \times 4 = 18.4</math></li> <li>• divide by 50 by dividing by 100 then doubling e.g. <math>270 \div 50 = 2.7 \times 2 = 5.4</math></li> </ul>

- **Multiplying and dividing by multiples of 10** - Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers.

Multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. $4.3 \times 10$ , $0.75 \times 100$ , $25 \div 10$ , $673 \div 100$
Divide a multiple of 10 by a single-digit number (whole number answers), e.g. $80 \div 4$ , $270 \div 3$
Multiply pairs of multiples of 10, and a multiple of 100 by a single digit number, e.g. $60 \times 30$ , $900 \times 8$
Multiply by 25 or 50, e.g. $48 \times 25$ , $32 \times 50$ using equivalent calculations, e.g. $48 \times 100 \div 4$ , $32 \times 100 \div 2$
Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6 kg to 2600 g, 3.5 cm to 35 mm, and 1.2 m to 120 cm
Multiply pairs of multiples of 10 and 100, e.g. $50 \times 30$ , $600 \times 20$
Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. $600 \div 20$ , $800 \div 400$ , $2100 \div 300$
Divide by 25 or 50
Convert between units of measurement using decimals to two places, e.g. change 2.75 l to 2750 ml, or vice versa

- **Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers** - Once children are familiar with some multiplication facts, they can use these facts to work out others. One strategy that can be used is writing one of the numbers as the sum of two others about which more is known:  $6 \times 7 = 6 \times (2 + 5) = 6 \times 2 + 6 \times 5$ . Another strategy is making use of factors, so  $7 \times 6$  is seen as  $7 \times 3 \times 2$ .

#### Distributive Law

Partition one of the numbers and use the distributive law of multiplication over addition.

$$6 \times 7 = 6 \times (5 + 2) = 6 \times 5 + 6 \times 2$$

Subtraction can be used similarly, so 'nine eights are ten eights minus one eight'.

#### Factors

$7 \times 6$  is seen as  $7 \times 3 \times 2$ .

#### Partitioning

$26 \times 3$  can be worked out by partitioning 26 into  $20 + 6$ , multiplying each part by 3, then recombining.

$$\begin{array}{r} 13 \times 12 \\ \swarrow \searrow \\ 3 \quad 4 \\ 13 \times 3 \times 4 \end{array}$$

$$\begin{array}{r} 13 \times 12 \\ \swarrow \searrow \searrow \\ 3 \quad 2 \quad 2 \\ 13 \times 3 \times 2 \times 2 \end{array}$$

$$\begin{array}{r} 13 \times 12 \\ \swarrow \searrow \\ 2 \quad 6 \\ 13 \times 2 \times 6 \end{array}$$

$$\begin{array}{r} 13 \times 12 \\ \swarrow \searrow \searrow \\ 2 \quad 2 \quad 3 \\ 13 \times 2 \times 2 \times 3 \end{array}$$

Multiply and divide two-digit numbers by 4 or 8, e.g. $26 \times 4$ , $96 \div 8$
Multiply two-digit numbers by 5 or 20, e.g. $32 \times 5$ , $14 \times 20$
Multiply by 25 or 50, e.g. $48 \times 25$ , $32 \times 50$
Multiply a two-digit and a single-digit number, e.g. $28 \times 7$
Divide a two-digit number by a single-digit number e.g. $68 \div 4$
Divide by 25 or 50, e.g. $480 \div 25$ , $3200 \div 50$
Find new facts from given facts, e.g. <ul style="list-style-type: none"> <li>given that three oranges cost 24p, find the cost of four oranges</li> </ul>

- given that three oranges cost 24p, find the cost of four oranges

- **Finding fractions, decimals and percentages** - Children need an understanding of how fractions, decimals and percentages relate to each other, e.g. if they know that  $\frac{1}{2}$ , 0.5 and 50% are all ways of representing the same part of a whole, then the calculations

- $\frac{1}{2} \times 40$
- $40 \times 0.5$
- 50% of £40

- can be seen as different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.

Recall percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths
Find fractions of whole numbers or quantities, e.g. $\frac{2}{3}$ of 27, $\frac{4}{5}$ of 70 kg
Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80
Recall equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $\frac{35}{100}$
Find half of decimals with units and tenths, e.g. half of 3.2
Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g

<p><b>Place Value</b></p> <p>937 + 100      1969 + 100      546 - 40</p> <p>1.7 + 0.05      40 000 - 500</p> <p>246 ÷ 1      100 x 217      0.4 ÷ 10</p> <p>1.68 x 100      100 x 100</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>435 - 30    979 + 100    3.005 + 6.12    2.15 + 0.05</p> <p>100 x 412    0.9 ÷ 10    1.28 x 100    50,000 - 500</p> <p>10 x 100</p> <p>Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?</p> <p>Circle two numbers that added together make 0.25</p> <p>0.05   0.23   0.2   0.5</p> <p>Circle two numbers that multiply together to equal 1 million</p> <p>200   2,000   5,000   50,000</p> <p>Write the number that is 5 less than 10 million</p> <p>Write the number that is one hundred thousand less than six million</p> <p>Round 124,531 to the nearest 10,000, 1,000, 100</p> <p><b>Partitioning and Bridging</b></p> <p>58 + 6      5 + 47      630 + 73      680 + 78</p> <p>560 + 89      8900 + 230</p> <p>74 - 7      97 - 8      320 - 50      2300 - 600</p> <p>3400 - 1700</p> <p>5 - 2.65      8.1 - 2.75      <math>1\frac{2}{5} + \frac{3}{10} =</math>      <math>1\frac{3}{10} - \frac{2}{5} =</math></p> <p>£3367.40 - £1021.23</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>4 - 1.15    <math>1\frac{4}{5} + \frac{3}{10}</math>    <math>1\frac{1}{4} + \frac{1}{3}</math>    <math>1\frac{1}{5} - \frac{1}{4}</math>    <math>\frac{3}{4} + \frac{7}{8} =</math></p> <p>5,756 + 8,643    936 + 285</p>	<p><b>Compensation</b></p> <p>56 + 8      72 + 9      56 - 8      72 - 9</p> <p>371 + 18      255 + 49      304 + 299</p> <p>673 - 99      854 - 398      3720 - 996</p> <p>0.71 + 0.09    0.56 + 0.08    0.34 - 0.09</p> <p>£1.17 + £0.39    £8.89 - £4.99</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>468 - 9      472 - 9      15.98 + 26.314</p> <p>12 - 6.01      15.4 - 8.88</p> <p><b>Redistribution - Equal sum</b></p> <p>56 + 8      72 + 9      371 + 18      255 + 49</p> <p>304 + 267</p> <p>£37.67 + £3.85    563 + 397    890,488 + 4,890</p> <p>229,899 + 31,321</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>89,994 + 7,643    936 + 285    89,994 + 7,643</p> <p><b>Redistribution - Same difference</b></p> <p>85 - 18      42 - 17      88 - 43      437 - 103</p> <p>819 - 504      532,525 - 9897</p> <p>£122.56 - £87.99    9.1 - 6.7    15.3 - 5.7</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>468 - 9      472 - 9      122,456 - 11,999</p> <p>4 - 1.15      12 - 6.01</p> <p>15.4 - 8.88    234,897 - 45,996</p>	<p><b>Think Partition for x and ÷</b></p> <p>32 x 4    29 x 2    122 x 4    4.6 x 2</p> <p>75 x 3    8.3 x 6    39 x 7</p> <p>3.3 x 7    5 x 49    4 x 198    96 x 0.3</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>15 x 6.1    24 x 3    1.52 x 6    7,505 ÷ 5</p> <p>17 x 1½</p> <p><b>Make links to doubling and halving</b></p> <p>50 x 28    86 x 50    500 x 70    18 x 2.5</p> <p>86 x 2.5    160 x 35    500 x 88    1.5 x 6.6</p> <p>0.5 x 120    4.5 x 2.2    15% x 346    75% x 220</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>15% x 440    <math>\frac{2}{5}</math> x 140    24 x 3</p> <p>20% of 1500    95% of 240</p> <p><b>Multiplying and dividing fractions</b></p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p><math>\frac{3}{5} + 3</math>    <math>\frac{2}{5} + 2</math>    <math>\frac{3}{4} + 2</math>    <math>\frac{2}{5} \times 140</math>    <math>\frac{1}{4} \times \frac{1}{8}</math></p>
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<p><b>Re-ordering and finding complements</b></p> <p>11 + 59      33 + 57      14 + 90 + 86</p> <p>290 + 310      1.15 + 2.55      0.8 + 0.26</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>1,034 + 586    2.15 + 0.05</p> <p>Circle two numbers that added together make 0.25</p> <p>0.05   0.23   0.2   0.5</p>	<p><b>x and ÷ by powers of 10</b></p> <p>10 x 53      87 x 10      1000 x 14      100 x 8.3</p> <p>100 x 0.41</p> <p>30 x 3      7 x 0.3      30 x 30      30 x 70</p> <p>567 ÷ 100    36 ÷ 10    0.5 ÷ 10    280 ÷ 4</p> <p>5600 ÷ 80    30 = □ ÷ 12    270 ÷ 9 = □ ÷ 0.9</p> <p>7 x 0.001    1.8 ÷ 0.1    3.25 ÷ 0.00001</p> <p>Circle two numbers that multiply together to equal 10 million</p> <p>200   2,000   5,000   50,000</p> <p><i>Examples from 2016 KS2 and Sample Papers</i></p> <p>1440 ÷ 12      630 ÷ 9      1,320 ÷ 12</p> <p>0.9 ÷ 10</p> <p>20% of 1,800    20% of 1500    7,505 ÷ 5</p> <p>95% of 240</p> <p>100 x 412      0.9 ÷ 10      1.28 x 100</p> <p>50,000 - 500    10 x 100</p> <p>Circle two numbers that multiply together to equal 1 million</p> <p>200   2,000   5,000   50,000</p>	
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